Problem 1.16

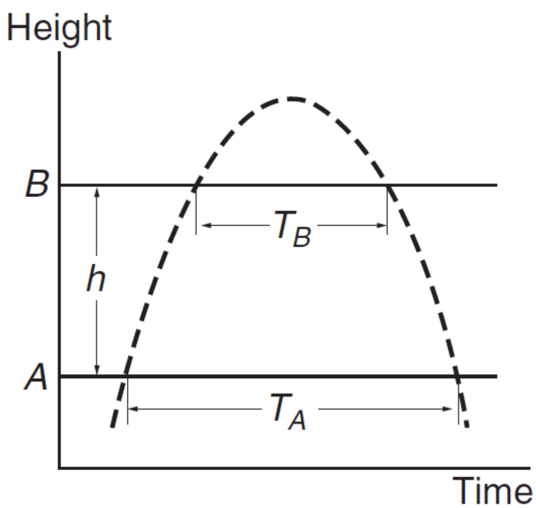
Measuring g

The acceleration of gravity can be measured by projecting a body upward and measuring the time that it takes to pass two given points in both directions.

Show that if the time the body takes to pass a horizontal line A in both directions is T_A, and the time to go by a second line B in both directions is T_B, then, assuming that the acceleration is constant, its magnitude is

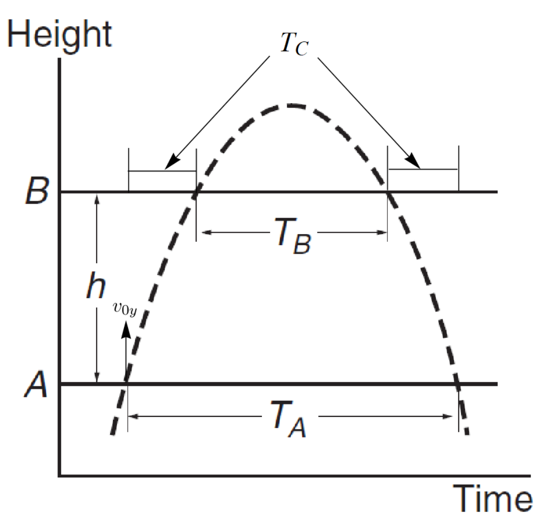
$$g = \frac{8h}{T_A^2 - T_B^2},$$

where h is the height of line B above line A.



Solution

Let T_C be the amount of time it takes for the body to go from A to B on the left side. Because there is no air resistance, it takes the body this same amount of time to go from B to A on the right side. Also, let v_{0y} be the y-component of the initial velocity at A on the left side.



The relationship between T_A, T_B, and T_C is

$$T_C + T_B + T_C = T_A.$$

$$\begin{aligned} 2T_C &= T_A - T_B \\ T_C &= \frac{T_A - T_B}{2}. \end{aligned} \tag{1}$$

Apply the kinematic formula,

$$v = v_0 + at,$$

to the body's motion going from A to the top of the parabola, where the velocity is zero.

$$\begin{aligned} 0 &= v_{0y} + (-g)\frac{T_A}{2} \\ v_{0y} &= \frac{g}{2}T_A \end{aligned} \tag{2}$$

Now the kinematic formula,

$$y = y_0 + v_0 t + \frac{1}{2}at^2,$$

will be applied to the body's motion going from A to B in order to relate h, v_{0y}, and T_C.

$$\begin{aligned} h &= 0 + v_{0y}T_C + \frac{1}{2}(-g)T_C^2 \\ &= v_{0y}T_C - \frac{g}{2}T_C^2 \\ &= \left(\frac{g}{2}T_A\right)\left(\frac{T_A - T_B}{2}\right) - \frac{g}{2}\left(\frac{T_A - T_B}{2}\right)^2 \\ &= \frac{g}{4}(T_A^2 - T_A T_B) - \frac{g}{2}\frac{T_A^2 - 2T_A T_B + T_B^2}{4} \\ &= \frac{g}{8}(2T_A^2 - 2T_A T_B - T_A^2 + 2T_A T_B - T_B^2) \\ h &= \frac{g}{8}(T_A^2 - T_B^2) \end{aligned}$$

Therefore, solving for g,

$$g = \frac{8h}{T_A^2 - T_B^2}.$$